TORSION

TORSION EQUATION

$$\frac{T}{I_P} = \frac{C\theta}{l} = \frac{f_s}{R}$$

Where T = Twisting moment

Ip = Polar Moment of Inertia

C = Modulus of Rigidty

Θ = Angle of twist in radians

l = length of shaft

fs = shear stress

R = radius of shaft

TORQUE TRANSMITTED BY SHAFT

$\frac{T}{I_P} = \frac{f_s}{R}$ from torsion equation

$T = Ip * \frac{f_s}{R}$

But $Ip = \frac{\pi}{32} * D^4$

$T = \frac{\pi}{32} * D^4 * \frac{f_s}{R}$

$T = \frac{\pi}{32} * D^4 * \frac{f_s * 2}{D}$

$\boldsymbol{T = \frac{\pi}{16} * fs * D^3}$

POWER TRANSMITTED BY SHAFT

$$P = \frac{2\pi N T_{mean}}{60} \quad \text{watts}$$

P = power transmitted in watts

N = r.p.m of the shaft

Tmean = Mean torque transmitted in N-m

TORSIONAL RIGIDITY

Torsional rigidity or stiffness of shaft is defined as the product of modulus of rigidity (C) and polar moment of inertia (Ip).

$$Torsional\ rigidity = C * I_P$$

But $\frac{T}{I_P} = \frac{C\theta}{l}$

$$C * I_P = \frac{T*l}{\theta}$$

$$Torsional\ rigidity = \frac{T*l}{\theta}$$

If l = 1meter and θ = 1 radian

Then torsional rigidity = Torque applied

STRENGTH OF A SHAFT

The strength of a shaft means the maximum torque or maximum power that can be safely transmitted by a shaft.

DIFFERENCE BETWEEN HOLLOW SHAFT AND SOLID SHAFT

- In the hollow shaft, the material at the centre is removed and spread at large radius. Therefore, hollow shafts are stronger than solid shaft having the same weight.
- The stiffness of the hollow shaft is more than the solid shaft with the same weight.
- The strength of the hollow shaft is more than the solid shaft with the same weight.
- The natural frequency of hollow shaft is higher than the solid shaft with the same weight.
- The hollow shaft is costlier than a solid shaft.
- The diameter of the hollow shaft is more than the solid shaft and require more space.
- Hollow shafts are having a more polar moment of inertia, thus they can transmit more torque compared to solid shafts.
- Hollow shafts do not transfer more power but the power to weight ratio of hollow shafts is more as compared to a solid shaft.
- Solid shafts, when subjected to bending, are stronger than that of a hollow shaft.

Numericals on TORSION

Problem 16.3. *In a hollow circular shaft of outer and inner diameters of 20 cm and 10 cm respectively, the shear stress is not to exceed 40 N / mm². Find the maximum torque which the shaft can safely transmit.*

Sol. Given :

Outer diameter, $D_0 = 20$ cm $= 200$ mm

Inner diameter, $D_i = 10$ cm $= 100$ mm

Maximum shear stress, $\tau = 40$ N/mm^2

Let T = Maximum torque transmitted by the shaft.

$$T = \frac{\pi}{16}\,\tau\left[\frac{D_0{}^4 - D_i{}^4}{D_0}\right] = \frac{\pi}{16} \times 40\left[\frac{200^4 - 100^4}{200}\right]$$

$$= \frac{\pi}{16} \times 40\left[\frac{16 \times 10^8 - 1 \times 10^8}{200}\right] = 58904860 \text{ Nmm}$$

$$= \mathbf{58904.86\ Nm.}$$ **Ans.**

Problem 16.4. *Two shafts of the same material and of same lengths are subjected to the same torque, if the first shaft is of a solid circular section and the second shaft is of hollow circular section, whose internal diameter is 2/3 of the outside diameter and the maximum shear stress developed in each shaft is the same, compare the weights of the shafts.*

(AMIE, Summer 1989)

Sol. Given :

Two shafts of the same material and same lengths (one is solid and other is hollow) transmit the same torque and develops the same maximum stress.

Let T = Torque transmitted by each shaft

τ = Max. shear stress developed in each shaft

D = Outer diameter of the solid shaft

D_0 = Outer diameter of the hollow shaft

D_i = Inner diameter of the hollow shaft = $\frac{2}{3} D_0$

W_s = Weight of the solid shaft

W_h = Weight of the hollow shaft

L = Length of each shaft

w = Weight density of the material of each shaft.

Torque transmitted by the solid shaft is given by equation (16.4)

π

$$T = \frac{\pi}{16} \tau D^3 \quad \ldots(i)$$

Torque transmitted by the hollow shaft is given by equation (16.6),

$$T = \frac{\pi}{16} \tau \left[\frac{D_0^4 - D_i^4}{D_0} \right] = \frac{\pi}{16} \tau \left[\frac{D_0^4 - (2/3\, D_0)^4}{D_0} \right]$$

$$= \frac{\pi}{16} \tau \left[\frac{D_0^4 - \frac{16}{81} D_0^4}{D_0} \right] = \frac{\pi}{16} \tau \times \frac{65\, D_0^4}{81 \times D_0}$$

$$= \frac{\pi}{16} \tau \times \frac{65\, D_0^3}{81} \quad \ldots(ii)$$

As torque transmitted by solid and hollow shafts are equal, hence equating equations (*i*) and (*ii*),

$$\frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \tau \times \frac{65}{81} D_0^3$$

Cancelling $\frac{\pi}{16} \tau$ to both sides

or $$D^3 = \frac{65}{81} D_0^3$$

$$\therefore \quad D = \left[\frac{65}{81} D_0^3\right]^{1/3} = \left(\frac{65}{81}\right)^{1/3} D_0 = 0.929\ D_0 \quad \text{...(iii)}$$

Now weight of solid shaft, W_s = Weight density × Volume of solid shaft

= w × Area of cross-section × Length

$$= w \times \frac{\pi}{4} D^2 \times L \quad \text{...(iv)}$$

Weight of hollow shaft,

W_h = w × Area of cross-section of hollow shaft × Length

$$= w \times \frac{\pi}{4} [D_0^2 - D_i^2] \times L = w \times \frac{\pi}{4} [D_0^2 - (2/3\ D_0)^2] \times L$$

$$= w \times \frac{\pi}{4}\left[D_0^2 - \frac{4}{9} D_0^2\right] \times L = w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L \quad \text{...(v)}$$

Dividing equation (*iv*) by equation (*v*),

$$\frac{W_s}{W_h} = \frac{w \times \frac{\pi}{4} D^2 \times L}{w \times \frac{\pi}{4} \times \frac{5}{9} D_0^2 \times L} = \frac{9D^2}{5D_0^2}$$

$$= \frac{9}{5} \times \frac{(0.929 D_0)^2}{D_0^2} \quad [\because \ D = 0.929\ D_0 \text{ from equation (iii)}]$$

$$= \frac{9}{5} \times 0.929^2 \times \frac{D_0^2}{D_0^2} = \frac{1.55}{1}$$

$$\therefore \quad \frac{\text{Weight of solid shaft}}{\text{Weight of hollow shaft}} = \frac{\mathbf{1.55}}{\mathbf{1}}. \quad \textbf{Ans.}$$

Problem 16.8. *Find the maximum shear stress induced in a solid circular shaft of diameter 15 cm when the shaft transmits 150 kW power at 180 r.p.m.*

Sol. Given :

Diameter of shaft, $D = 15 \text{ cm} = 150 \text{ mm}$

Power transmitted, $P = 150 \text{ kW} = 150 \times 10^3 \text{ W}$

Speed of shaft, $N = 180 \text{ r.p.m.}$

Let τ = Maximum shear stress induced in the shaft

Power transmitted is given by equation (16.7) as

$$P = \frac{2\pi NT}{60}$$

$$150 \times 10^3 = \frac{2\pi \times 180 \times T}{60}$$

$$\therefore \quad T = \frac{150 \times 10^3 \times 60}{2\pi \times 180} = 7957.7 \text{ Nm} = 7957700 \text{ Nmm}$$

Now using equation (16.4) as,

$$T = \frac{\pi}{16} \tau D^3$$

$$7957700 = \frac{\pi}{16} \times \tau \times 150^3$$

$$\therefore \quad \tau = \frac{16 \times 7957700}{\pi \times 150^3} = 12 \text{ N/mm}^2. \quad \textbf{Ans.}$$

REVISION

$$\frac{\boldsymbol{T}}{\boldsymbol{I_P}} = \frac{\boldsymbol{C\theta}}{\boldsymbol{l}} = \frac{\boldsymbol{f_s}}{\boldsymbol{R}}$$

(a) For solid shaft

$$\frac{T}{I_P} = \frac{f_s}{R}\ from\ torsion\ equation$$

$$T = Ip * \frac{f_s}{R}$$

$$But\ Ip = \frac{\pi}{32} * D^4$$

$$T = \frac{\pi}{32} * D^4 * \frac{f_s}{R}$$

$$T = \frac{\pi}{32} * D^4 * \frac{f_s * 2}{D}$$

$$\boldsymbol{T = \frac{\pi}{16} * fs * D^3}$$

(b) For hollow shaft

$$\frac{T}{I_P} = \frac{f_s}{R}\ from\ torsion\ equation$$

$$T = Ip * \frac{f_s}{R}$$

$$But\ Ip = \frac{\pi}{32} * (D^4 - d^4)$$

$$T = \frac{\pi}{32} * (D^4 - d^4) * \frac{f_s}{R}$$

$$T = \frac{\pi}{32} * (D^4 - d^4) * \frac{f_s * 2}{D}$$

$$\boldsymbol{T = \frac{\pi}{16} * fs * \frac{(D^4 - d^4)}{D}}$$

Find the torque which a shaft of 250 mm diameter can safely transmit if the stress does not exceed 46 N/mm^2.

Solution

$$T = \frac{\pi}{16} * f_s * D^3$$

$$T = \frac{\pi}{16} * 46 * 250^3$$

$$\boldsymbol{T = 141126232.7\ N - mm}$$

Calculate the length of wire of 10mm diameter so that it can be twisted through a complete revolution without exceeding shear stress more than 40 N/mm^2. Take G = 2.7 * 10^4 N/mm^2

Solution Diameter of wire, D = 10 mm

Angle of twist, θ = 1 revolution = 2π radians

Maximum shear stress, fs = 40 N/mm^2

Radius of wire = D/2 = 10/2 = 5 mm

$$\frac{f_S}{R} = \frac{G * \theta}{l}$$

$$l = \frac{G * \theta * R}{f_S}$$

$$l = \frac{2.7 * 10^4 * 2\pi * 5}{40}$$

$$\boldsymbol{l = 21205.75\ mm = 21.205\ m}$$

In a hollow circular shaft of outer and inner diameter of 200 mm and 100 mm respectively, the shear stress is not to exceed 40 N/mm^2. Find the maximum torque which the shaft can safely transmit.

Solution Outer Diameter D = 200 mm

Inner diameter, d = 100 mm

Shear stress, fs = 40 N/mm^2

$$T = \frac{\pi}{16} * fs * \frac{(D^4 - d^4)}{D}$$

$$T = \frac{\pi}{16} * 40 * \frac{(200^4 - 100^4)}{200}$$

$$\boldsymbol{T = 58904862.25\ N - mm}$$

$$\boldsymbol{T = 58904.86\ N - m}$$

A solid shaft of 200mm diameter is replaced by a hollow shaft having internal diameter 150mm. Calculate the external diameter of hollow shaft is cross-section area of both shaft is same.
(a) Compare the strength of both shafts if the angle of twist is same for both shafts.
(b) Compare the angle of twist of both shafts if length of each shaft and shear stress produced in both shafts is same.

Solution

Diameter of solid shaft D_1 = 200mm
Internal Diameter of shaft, d = 150mm
Outer diameter of shaft = D
Since the area of cross-section of both shafts is same.

$$\frac{\pi}{4}D_1^2 = \frac{\pi}{4}(D^2 - d^2)$$

$$\frac{\pi}{4}200^2 = \frac{\pi}{4}(D^2 - 150^2)$$

$$D^2 = 400 * 10^2 + 225 * 10^2$$

$$D^2 = 62500$$

$$D = 250mm$$

(a) Comparison of Strength

For solid shaft: From torsion equation

$$\frac{T_s}{I_{ps}} = \frac{f_{ss}}{R_s}$$

$$T_s = \frac{f_{ss}}{R_s} * I_{ps}$$

$$T_s = \frac{f_{ss} * \frac{\pi}{32}D_1^4}{D_1/2}$$

$$T_s = f_{ss} * \frac{\pi}{16}D_1^3$$

$$\boldsymbol{T_s = f_{ss} * \frac{\pi}{16}200^3}$$

For hollow shaft

$$\frac{T_H}{I_{PH}} = \frac{f_{SH}}{R_H}$$

$$T_H = \frac{f_{SH}}{R_H} * I_{PH}$$

$$T_H = \frac{f_{SH} * \frac{\pi}{32}(D^4 - d^4)}{D/2}$$

$$\boldsymbol{T_H = f_{SH} * \frac{\pi}{16}\left(\frac{250^4 - 150^4}{250}\right)}$$

Now

$$\boldsymbol{\frac{T_H}{T_S} = \frac{f_{SH} * \frac{\pi}{16}\left(\frac{250^4 - 150^4}{250}\right)}{f_{ss} * \frac{\pi}{16}200^3}}$$

$$But\ f_{SH} = f_{SS}$$

$$\frac{T_H}{T_S} = \frac{136 * 10^5}{8 * 10^6}$$

$$\boldsymbol{\frac{T_H}{T_S} = 1.7}$$

(b) Comparison of angle of twist

Let θ_S is the angle of twist for solid shaft and θ_H is the angle of twist for hollow shaft.

From Torsion equation

$$\frac{f_{SS}}{R_S} = \frac{G_S * \theta_S}{l_S}$$

$$\theta_S = \frac{f_{SS} * l_S}{G_S * R_S}$$

$$\theta_H = \frac{f_{SH} * l_H}{G_H * R_H}$$

$$\frac{\theta_H}{\theta_S} = \frac{\frac{f_{SH} * l_H}{G_H * R_H}}{\frac{f_{SS} * l_S}{G_S * R_S}}$$

$$f_{SH} = f_{SS} \text{ and } G_S = G_H$$

$$\boldsymbol{\frac{\theta_H}{\theta_S} = \frac{R_S}{R_H}}$$

Diameter of the solid shaft = 200 mm and R_S = 100 mm

Diameter of the hollow shaft = 250 mm and R_H = 125 mm

$$\boldsymbol{\frac{\theta_H}{\theta_S} = \frac{100}{125} = 0.8}$$

A solid shaft is subjected to a torque of 15000 N-m. Find the necessary diameter of the shaft if the allowable shear stress is 60 N/mm^2. The allowable twist 1° for every 20 diameters length of the shaft. Take G = $0.8*10^5$ N/mm^2.

Solution

T = 15000 N-m = 15000 * 1000 N-mm or $15*10^6$ N-mm

fs = 60 N/mm^2

θ = 1° = π/180 radian

Diameter of shaft = D

Length of shaft l = 20D

(a) Considering allowable shear stress

$$T = \frac{\pi}{16} * f_s * D^3$$

$$15 * 10^6 = \frac{\pi}{16} * 60 * D^3$$

$$D^3 = 1273239.545$$

$$\boldsymbol{D = 108.4\ mm}$$

(b) Considering allowable twist

$$\frac{T}{I_P} = \frac{G\theta}{l}$$

$$But\ we\ know\ I_P = \frac{\pi}{32} * D^4$$

$$\frac{15 * 10^6}{\frac{\pi}{32} * D^4} = \frac{0.8 * 10^5 * \pi}{180 * 20D}$$

$$D^3 = 2188537.567$$

$$\boldsymbol{D = 130\ mm}$$

Suitable diameter of the shaft is 130 mm

A hollow shaft is to transmit 250kW at 80 rpm. If the shear stress is not to exceed 60 N/mm^2 and the internal diameter is 0.6 of the external diameter. Find the external and internal diameter assuming that the maximum torque is 30% more than mean torque.

Solution

Power = 250*10^3 Watt

N = 80 rpm

D = External diameter of the shaft

D = Internal diameter of the shaft = 0.6*D

$$P = \frac{2\pi N T_{mean}}{60}$$

$$250000 = \frac{2 * \pi * 80 * T_{mean}}{60}$$

$$T_{mean} = 29841.55\ N - m$$

$$T_{mean} = 29841550\ N - mm$$

$$Given\ that\ \boldsymbol{T_{max}} = \boldsymbol{1.3} * \boldsymbol{T_{mean}}$$

$$T_{max} = 1.3 * 29841550 = 38794015\ N - mm$$

$$T = \frac{\pi}{16} * f_s * \left(\frac{D^4 - d^4}{D}\right)$$

$$38794015 = \frac{\pi}{16} * 60 * \left(\frac{D^4 - (0.6D)^4}{D}\right)$$

$$38794015 = \frac{\pi}{16} * 60 * \left(\frac{D^4 - 0.13^4}{D}\right)$$

$$D^3 = 3784986.5$$

$$\boldsymbol{D = 156\ mm}$$

A solid shaft of 150 mm diameter is to transmit 180 kW at 100 rpm. Determine the angle of twist for a length of 5 m, if the maximum torque exceeds the mean by 40%. Take G = $8*10^4$ N/mm^2.

Solution

D = 150 mm

Power, P = 180kW = 180000 W

Speed of the shaft, N = 100 rpm

Length of shaft, l = 5 m = 5000 mm

$$P = \frac{2\pi N T_{mean}}{60}$$

$$180000 = \frac{2 * \pi * 100 * T_{mean}}{60}$$

$$T_{mean} = 17188.734\ N - m$$

$$T_{mean} = 17188734\ N - mm$$

$$Given\ that\ \boldsymbol{T_{max} = 1.4 * T_{mean}}$$

$$T_{max} = 1.4 * 17188734 = 24064227.6\ N - mm$$

$$But\ we\ know\ I_P = \frac{\pi}{32} * D^4$$

$$I_P = \frac{\pi}{32} * 150^4 = 49700977.53\ mm^4$$

$$\frac{T}{I_P} = \frac{G\theta}{l}$$

$$\frac{24064227.6\ N}{49700977.53} = \frac{8 * 10^4 * \theta}{5000}$$

$$\theta = 0.03\ radians$$

$$\boldsymbol{\theta = 0.03 * \frac{180}{\pi} = 1.72°}$$