SPRINGS

INTRODUCTION

Spring is an elastic machine element that can deflect under the application of load.

When the load is removed, it regains its original position.

In other words, spring is a mechanical object made up of material having very high yield strength to restore elastic.

It is used in various machines to absorb shocks or it also resist to transfer shocks and vibrations on various critical machine members.

SPRING MATERIALS

The material used to make springs are called a spring steel.

Spring steels are mostly low-alloy manganese, low carbon steel or high carbon steel with very high yield strength.

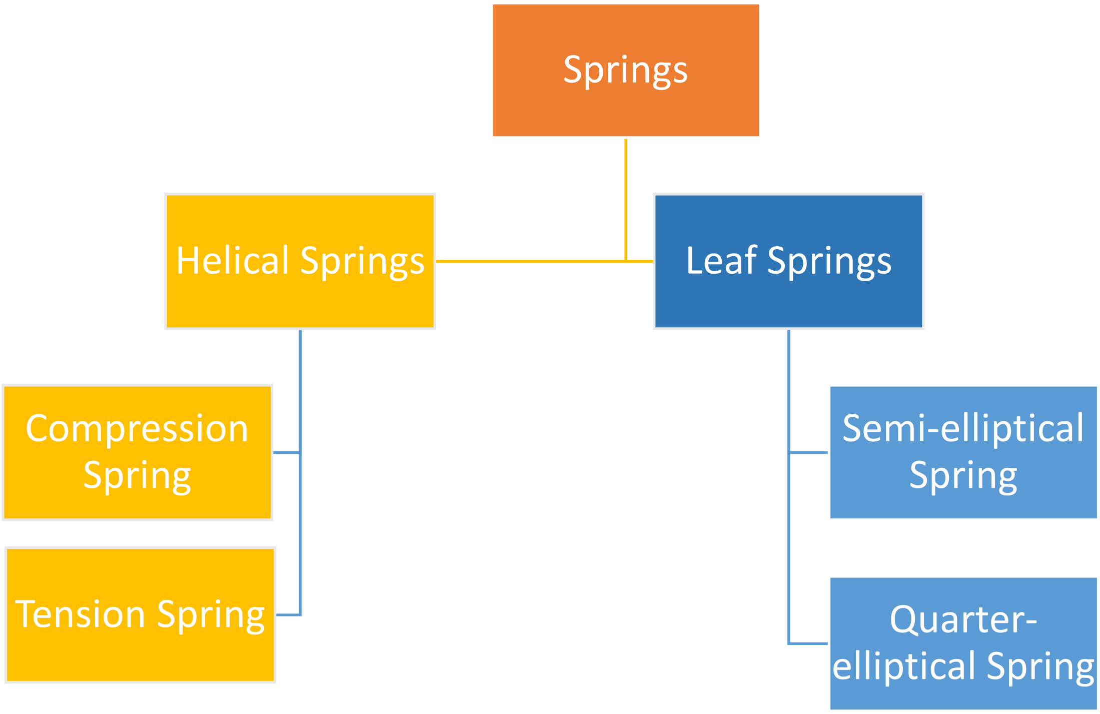
Examples of spring materials are as follows:

1) Oil Tempered Steel
2) Stainless Steel
3) Elgiloy
4) Carbon Value
5) Iconel
6) Monel
7) Titanium
8) Chrome Silicon

USES OF SPRINGS

1) To absorb shock load
2) To store energy
3) To measure force
4) To motive power
5) To Return motion
6) To control of vibrations
7) To retaining of rings

TYPES OF SPRINGS



Helical Springs

Compression spring (Open coiled spring)

They are designed to operate in compression load. Compression springs helical coils are spaced at relatively larger distance. When a compression force is applied, their length reduces. Compression springs application includes:

- Shock absorber
- Suspension
- Retractable Pen etc.

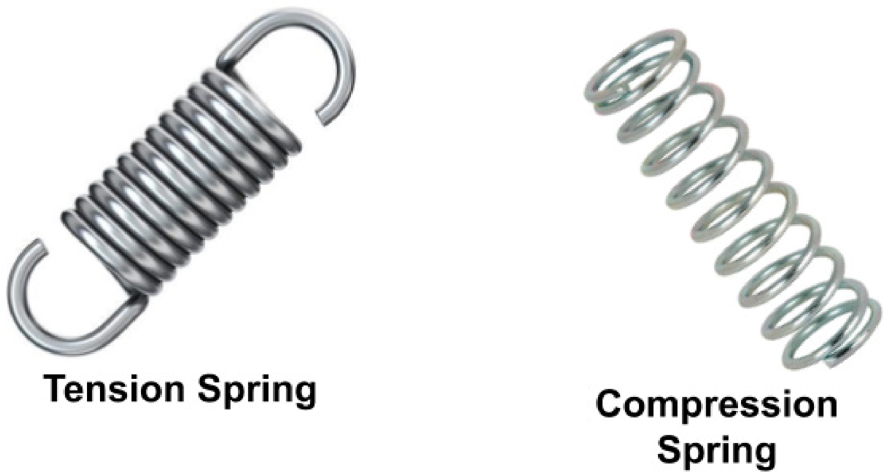


Figure 1 Types of Helical Springs

Tension Springs (Closed coiled spring)

Tension helical springs are designed to operate in tension loads. In this the spring coils are spaced at very small distance.

- Garage Door Mechanism
- Weighing Machine.
- Spring Loaded jaw Pliers etc.

Leaf Spring

They are also known as laminated / carriage / semi-elliptical springs.

Leaf spring consists of a number of flat plates also known as leaves of varying length sandwiched one on another using clamps and bolt.

Leaf spring consists of a number of leaves, made of steel plates, of increasing lengths from the centre. All the leaves are clamped by a centre bolt at the centre and side almost at the sides so that the leaves are in position.

The main leaf is the longer one having bent ends, called the spring eyes. The spring eye is connected to the frame by a shackle. The centre portion of the spring is connected to the front axle by U-bolt.

Figure 2 Leaf Spring

Semi-elliptical Springs

Semi-elliptical springs are usually used in all the vehicle. Particularly in trucks, semi-elliptical springs are fitted in front and rear axles.

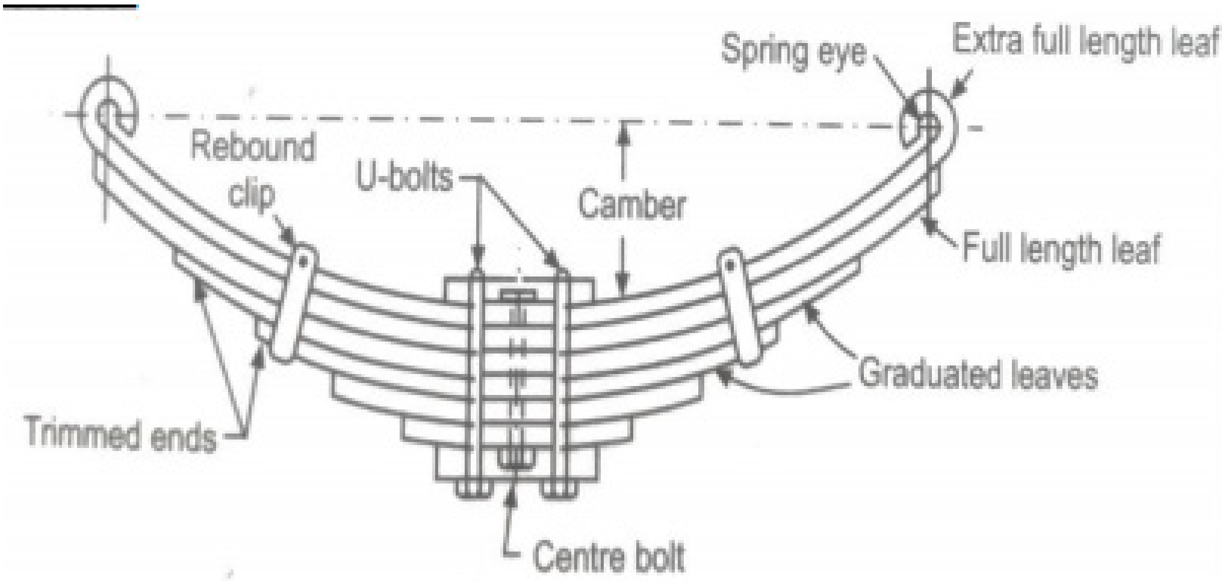


Figure 3 Semi-elliptical Spring

But in cars, they are fitted on the rear axle only and the independent suspension is fitted on the front axle. Semi-elliptical springs are cheaper and require less repairing. They increase the range of spring action and last for a long time.

Quarter-elliptical Spring

Quarter-elliptical springs were used in old small cars, like Chrysler cars. This type of spring consists only a quarter portion of the full elliptical spring and fitted with the frame by the bolt.

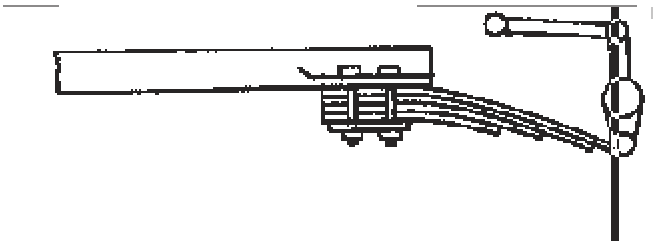


Figure 4 Quarter-elliptical Spring

Terms used in Compression Springs

1. *Solid length*: When the compression spring is compressed until the coils come in contact with each other, then the spring is said to be solid. The solid length of a spring is the product of total number of coils and the diameter of the wire.

   Mathematically,

   Solid length of the spring, **LS = n * d**

   where n = Total number of coils, and d = Diameter of the wire.

2. *Free length:* The free length of a compression spring, as shown in Fig., is the length of the spring in the free or unloaded condition. It is equal to the solid length plus the maximum deflection or compression of the spring and the clearance between the adjacent coils (when fully compressed).
3. *Spring index:* The spring index is defined as the ratio of the mean diameter of the coil to the diameter of the wire.

   Mathematically,

   Spring index, **C = D / d**

   where D = Mean diameter of the coil, and d = Diameter of the wire.

4. *Spring rate:* The spring rate (or stiffness or spring constant) is defined as the load required per unit deflection of the spring.

   Mathematically,

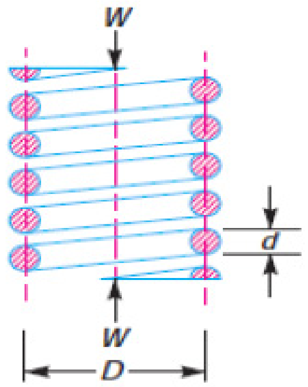
   Spring rate, **k = W / δ**

   where W = Load, and δ = Deflection of the spring.

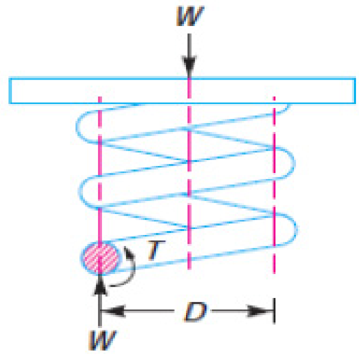
5. *Pitch:* The pitch of the coil is defined as the axial distance between adjacent coils in uncompressed state.

Stress in Helical Springs of circular wire

Consider a helical compression spring made of circular wire and subjected to an axial load W, as shown in Fig.



(a) Axially loaded helical spring.



(b) Free body diagram showing that wire is subjected to torsional shear and a direct shear.

Let D = Mean diameter of the spring coil,

d = Diameter of the spring wire,

n = Number of active coils,

G = Modulus of rigidity for the spring material,

W = Axial load on the spring,

τ = Maximum shear stress induced in the wire,

C = Spring index = D/d,

p = Pitch of the coils, and

δ = Deflection of the spring, as a result of an axial load W

Now consider a part of the compression spring as shown in Fig (b). The load W tends to rotate the wire due to the twisting moment (T) set up in the wire. Thus, torsional shear stress is induced in the wire.

A little consideration will show that part of the spring, as shown in Fig (b), is in equilibrium under the action of two forces W and the twisting moment T. We know that the twisting moment,

$$T = W * \frac{D}{2}$$

$$\text{Also } T = \frac{\pi}{16} * \tau * d^3$$

$$W * \frac{D}{2} = \frac{\pi}{16} * \tau * d^3$$

$$\boldsymbol{\tau = \frac{8WD}{\pi d^3}}$$

Strain Energy (U)

When Subjected to Axial Load

Neglecting strain energy due to direct shear W, strain energy stored

$$U = \frac{\tau^2}{4G} * volume\ of\ spring$$

Substitute value of τ from above

$$U = \frac{\left(\frac{8WD}{\pi d^3}\right)^2}{4G} * \frac{\pi d^2}{4}(\pi d n)$$

$$\boldsymbol{U = \frac{4W^2D^3n}{Gd^4}}$$

Deflection of Springs

Let δ = Deflection of the spring, as a result of an axial load W

Then, work done by the load $= \frac{1}{2} * W * \delta$

Equating the work done to the strain energy stored in the spring

$$\frac{1}{2} * W * \delta = \frac{4W^2D^3n}{Gd^4}$$

$$\boldsymbol{\delta = \frac{8WD^3n}{Gd^4}}$$

$$\boldsymbol{\delta = \frac{64WR^3n}{Gd^4}}$$

Stiffness of Springs (k)

The stiffness of the spring is defined as the load required to produce unit deflection.

$$Stiffness\ of\ spring(k) = \frac{W}{\delta}$$

$$k = \frac{W}{\frac{64WR^3n}{Gd^4}}$$

$$\boldsymbol{k = \frac{Gd^4}{64R^3n}}$$

COMPOUND SPRINGS

More than one spring may have, at times, to be used to meet the specific requirements. The springs may be used either side by side (in parallel) or connected end to end (in series) as shown in Figures 5 (a) and (b). This type of composite system of springs is called as compound springs.

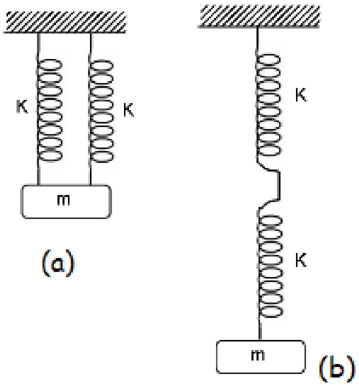


Figure 5 Springs connected in Parallel (a) and Series (b)

Springs in Parallel

Figure 5 (a) shows two springs connected in parallel.

In this case,

a. The extension of both springs is equal.
b. The sum of the load carried by each spring is equal to the load W.

$$W = W_1 + W_2$$

W_1 = load shared by the spring 1, and W_2 = load shared by the spring 2.

$$\delta S = \delta S_1 + \delta S_2$$

$$\boldsymbol{S = S_1 + S_2}$$

Springs in Series

Figure 5 (b) shows two springs connected in series.

Each spring carries the same load W applied at the end and the total deflection is equal to the sum of the deflection in each spring.

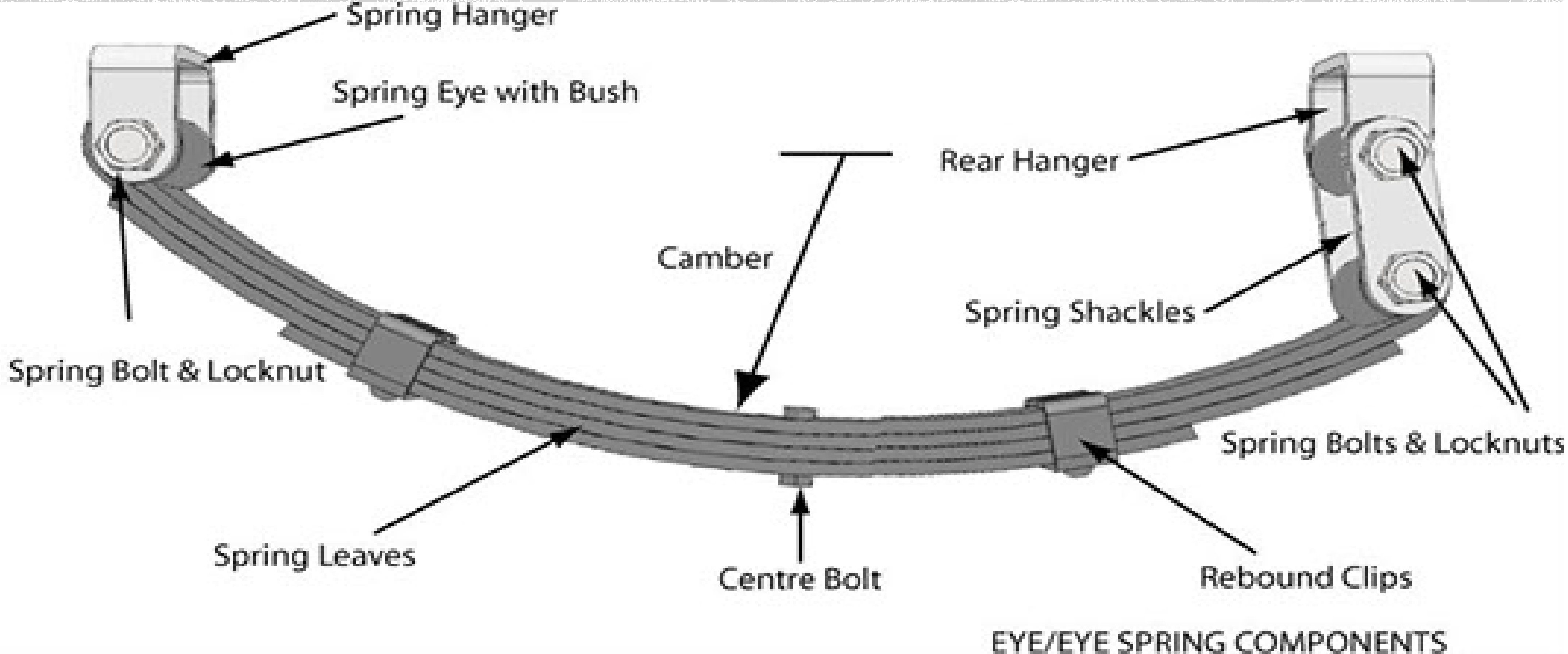
$$\delta = \delta_1 + \delta_2$$

$$but\ we\ know \quad S = \frac{W}{\delta}$$

$$\frac{W}{S} = \frac{W}{S_1} + \frac{W}{S_2}$$

$$\boldsymbol{\frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2}}$$

LEAF SPRINGS



INTRODUCTION TO LEAF SPRINGS

- This type of springs are commonly used in carriages such as cars, railway wagons etc. and they are also termed as laminated or carriage springs.

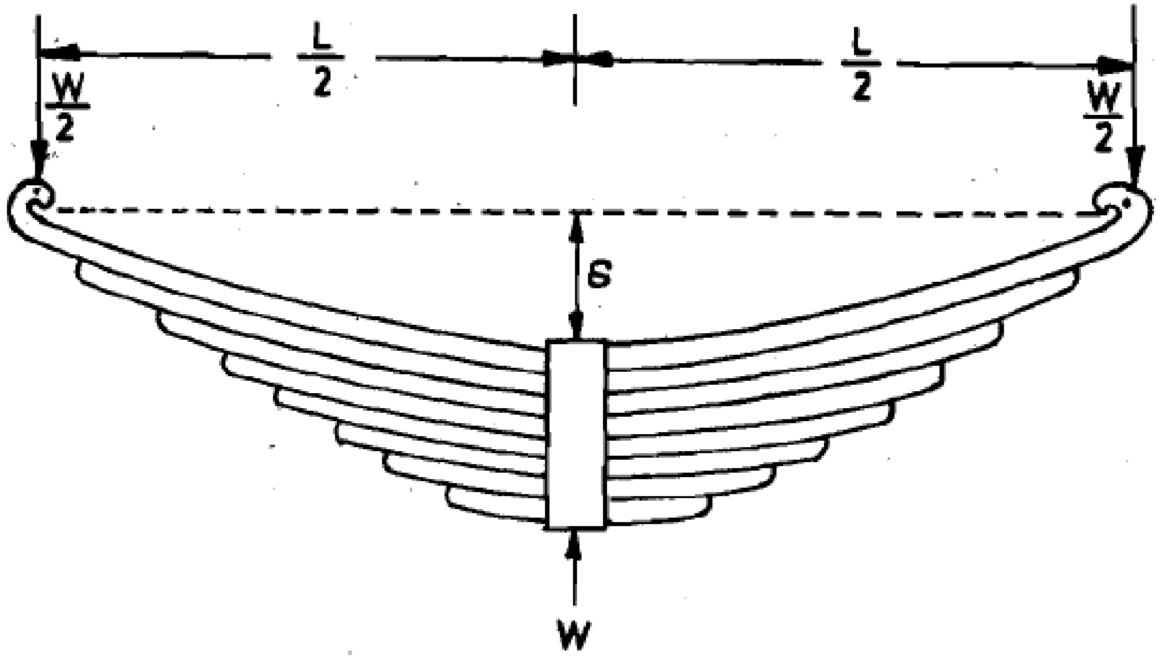
- It is made up of a number of leaves of equal width and thickness, but varying length placed in laminations and loaded as a beam.

- The lengths of the plates are so adjusted that the maximum bending stress remains same in every plate and thereby it behaves like a beam of uniform strength.

STRESS IN SPRINGS

- Figure shows a carriage spring carrying a central vertical load W, which is balanced by equal end reactions -W/2

W = load on the spring
R = initial radius of curvature of plates
δ = initial central deflection
b = width of each plate
t = thickness of each plate
n = number of leaves (plates)
L = span of the spring
fb = bending stress



- Section modulus for a single plate or laminate $= \frac{bt^2}{6}$
- Section modulus for the whole spring (having n laminates) $= \mathrm{n} * \frac{bt^2}{6}$
- Maximum bending moment, M $= \frac{WL}{4}$
- But $\frac{M}{I} = \frac{f}{y}$
- or $M = f * Z$
- $\frac{WL}{4} = \mathrm{f} * (\mathrm{n} * \frac{bt^2}{6})$
- $f = \frac{3}{2} * \frac{WL}{nbt^2}$

STRAIN ENERGY

- Total strain energy $U = \frac{f^2}{6E} * (volume\ of\ equivalent\ plate)$
- Volume $= \frac{nb}{2} Lt$
- Strain Energy $U = \frac{f^2}{6E} * \frac{nb}{2} Lt$
- Substitute value of ‘f’
- $U = \frac{(\frac{3}{2} * \frac{WL}{nbt^2})^2}{6E} * \frac{nb}{2} Lt$
- Work done by the load $= \frac{1}{2} * W * \delta$
- Equating these two i.e. U = Work done

contd.

- $\frac{1}{2} * W * \delta = \frac{(\frac{3}{2} * \frac{WL}{nbt^2})^2}{12E} * nbLt$

- $\delta = \frac{3WL^3}{8Enbt^3}$

STIFFNESS OF SPRINGS

- It is defied as tile load required to produce unit deflection
- Spring constant $S = \frac{W}{\delta}$
- $S = \frac{8Enbt^3}{3L^3}$

PRACTICAL APPLICATIONS

- Leaf springs are extensively used in railway carriages, railway wagons, trucks, trollies, buses and cars etc.
- The common purpose of all kinds of springs is to absorb energy and to release it as and when required.
- Carriage springs are used normally to absorb shock. In other words, they act as primarily shock absorbers.

Numericals based on Helical & Leaf Springs

A close coiled helical spring is made of 5 mm diameter wire. It is made up of 30 coils, each of mean diameter 75 mm. If the maximum stress in the spring is not to exceed 200 MPa, then determine

(a) the proof load

(b) the extension of the spring when carrying this load.

Take $G = 80$ GPa.

Solution

Here, we have $d = 5$ mm, $n = 30$

$D = 75$ mm, $R = 37.5$ mm

$(f_s)_{max} = 200$ MPa, $G = 80$ GPa

$$\therefore \quad W = \frac{\pi d^3}{8D} f_s$$

Thus, proof load $= \frac{\pi (5)^3}{8 \times 75} \times 200 = 131$ N

Deflection $\delta = \frac{64\, W R^3 n}{G d^4}$

$$\delta = \frac{64 \times 131 \times (37.5)^3 \times 30}{(80 \times 10^3)\,(5)^4} = 265.5 \text{ mm}$$

A helical spring in which the slope of the helix may be assumed small, is required to transmit a maximum pull of 1 kN and to extend 10 mm for 200 N load. If the mean diameter of the coil is to be the 80 mm, find the suitable diameter for the wire and number of coils required. Take G = 80 GPa and allowable shear stress as 100 MPa.

Shear stress, $f_s = \frac{8WD}{\pi d^3}$

$$\therefore \quad d^3 = \frac{8WD}{\pi f_s} = \frac{8 \times 1000 \times 80}{\pi \times 100}$$

Here, we have $W = 1000$ N D = 80 mm

$f_s = 100$ MPa

∴ Diameter of spring wire = 12.68 mm.

Now $\delta = 10$ mm for $W = 200$ N.

$$\therefore \text{ Spring constant, } k = \frac{W}{\delta} = \frac{200}{10} = 20 \text{ N/mm}$$

$$= \frac{20}{10^{-3}} = 2 \times 10^4 \text{ N/m}$$

We have, $C = 80$ GPa

$$\delta = \frac{8WD^3 n}{Gd^4}$$

$$\therefore \quad n = \frac{Gd^4}{8\left(\frac{W}{\delta}\right)D^3} = \frac{(80 \times 10^3)(12.68)^4}{8 \times 20 \times (80)^3}$$

$$= 25.28$$

Number of coils required = 25.28 say 26.

A leaf spring 0.8 m long consists of 12 plates, each of them is 65 mm wide and 6 mm thick. It is simply supported at its ends. The greatest bending stress is not to exceed 180 MPa and the central deflection when the spring is fully loaded is not to exceed 20 mm. Estimate the magnitude of the greatest central load that can be applied to the spring. Take $E = 200 \times 10^3$ MPa.

Here, we have, $L = 0.8$ m, $n = 12$

$b = 65$ mm, $t = 6$ mm

$f \leq 210$ N/mm^2, $\delta \leq 20$ mm

Using the relationship, $f_{max} = \dfrac{3WL}{2nbt^2}$

$$180 = \frac{3W \times 800}{2 \times 12 \times 65 \times (6)^2}$$

$$\therefore \quad W = 4212 \text{ N}$$

Using the relationship, $\delta = \dfrac{3}{8} \times \dfrac{WL^3}{Enbt^3}$

$$20 = \frac{3}{8} \times \frac{W \times (800)^3}{(200 \times 10^3) \times 12 \times (65) \times (6)^3}$$

$$\therefore \quad W = 3510 \text{ N}$$

Thus, the greatest central load that can be applied is lesser of these two, i.e. 3.51 kN.

A leaf spring is required to satisfy the following specification :

$L = 0.75$ m, $W = 5$ kN, $b = 75$ mm, maximum stress = 210 MPa,

Maximum deflection = 25 mm, $E = 200$ GPa.

Find the number of leaves, their thicknesses and initial radius of curvature.

Here, we have, $L = 0.75 \text{ m} = 750 \text{ mm}$ $W = 5 \text{ kN} = 5000 \text{ N}$

$\delta \leq 25 \text{ mm}$ $f \leq 210 \text{ N/mm}^2$

$$\text{Maximum stress, } f = \frac{3}{2} \times \frac{WL}{nbt^2}$$

$$210 = \frac{3}{2} \times \frac{5000 \times 750}{75 \times nt^2}$$

$$\therefore \quad nt^2 = 357.2 \qquad \text{(i)}$$

$$\text{Maximum deflection, } \delta = \frac{3}{8} \times \frac{WL^3}{Enbt^3}$$

$$25 = \frac{3}{8} \times \frac{5000 \times 750^3}{200 \times 10^3 \times (75) \times nt^3}$$

$$\therefore \quad nt^3 = 2109 \qquad \text{(ii)}$$

From (i) and (ii), $t = 5.905$ mm

Thus, use 6 mm thick plates.

$$\therefore \quad n = \frac{357.2}{6^2} = 9.916$$

Adopt 10 leaves.