Unit - Integral Calculus.

Ch-1 Indefinite Integral

Pg IC-2,3

$$\frac{d}{dx} F(x) = f(x)$$

$$F(x) = \int f(x)\,dx$$

or $$\int f(x)\,dx = F(x)$$

$$\int 1\,dx = x + c$$

$$\int 0\,dx = c$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$$

$$\int \frac{1}{x}\,dx = \log x + c$$

$$\int e^x = e^x + c$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

$$\int \sec^2 x\,dx = \tan x$$

$$\int \operatorname{cosec}^2 x\,dx = -\cot x$$

$$\int \sec x.\tan x\,dx = \sec x.$$

$$\int \operatorname{cosec} x.\cot x\,dx = -\operatorname{cosec} x$$

$$\int a^x = \frac{a^x}{\log a} + c$$

Unit Integral Calculus

Ch-1 Indefinite Integral

Q $\int x^4\,dx$

Sol $\dfrac{x^{4+1}}{4+1} + c$

$= \dfrac{x^5}{5} + c$

Q $\int e^{-mx}\,dx$

Sol $\dfrac{e^{-mx}}{-m} + c$

$= -\dfrac{e^{-mx}}{m} + c$

$\int x^n\,dx = \dfrac{x^{n+1}}{n+1}$, $\int e^x = e^x + c$

Q $\int \cos^2 x\,dx$ Pg IC-11

Sol $\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \dfrac{\cos 2x + 1}{2}$

or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

$= \frac{1}{2}\int (1 + \cos 2x)\,dx$

$= \frac{1}{2}\left[\int 1\,dx + \int \cos 2x\,dx\right]$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]$

$= \frac{x}{2} + \frac{\sin 2x}{4} + c$

Unit - Integral Calculu

Ch-1 Indefinite I

Q $\int \frac{dx}{1+\sin x} dx$

Sol Rationalising the den.

$$\int \frac{dx}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}$$

$$= \int \frac{1-\sin x}{1-\sin^2 x} dx$$

$$= \int \frac{1-\sin x}{\cos^2 x} dx$$

$$= \int \frac{1}{\cos^2 x} dx - \int \frac{\sin x}{\cos x . \cos x} dx$$

$$= \int \sec^2 x - \int \tan x \cdot \sec x$$

$$= \tan x - \sec x + c$$

Ch-1 Indefinite Integral

Q $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}$

Sol $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}} \times \frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}-\sqrt{x}}\,dx$

$$= \int \frac{\sqrt{x+1}-\sqrt{x}}{\cancel{x}+1-\cancel{x}}\,dx$$

$$= \int \sqrt{x+1}\,dx - \int \sqrt{x}\,dx$$

$$= \int (x+1)^{1/2}\,dx - \int (x)^{1/2}\,dx$$

$$= \frac{(x+1)^{3/2}}{\frac{3}{2}(1)} - \frac{x^{3/2}}{\frac{3}{2}} = \frac{2}{3}\left[(x+1)^{3/2} - x^{3/2}\right] + c$$

$$\int (ax+b)^n = \frac{(ax+b)^{n+1}}{(n+1)\,\underline{(a)}}$$

Unit - Integral Calculus.

Ch - 1 Indefinite Integral

Q $\int \sqrt{1 + \sin x}\, dx$

Sol $\int \sqrt{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2} + 2 \sin \frac{x}{2} \cos \frac{x}{2}}\, dx$

$= \int \sqrt{\left(\sin \frac{x}{2} + \cos \frac{x}{2}\right)^2}\, dx$

$= \int \left(\sin \frac{x}{2} + \cos \frac{x}{2}\right) dx$

$= \int \sin \frac{x}{2}\, dx + \int \cos \frac{x}{2}\, dx$

$= \dfrac{-\cos \frac{x}{2}}{\frac{1}{2}} + \dfrac{\sin \frac{x}{2}}{\frac{1}{2}} + c$

$= 2\left[\sin \frac{x}{2} - \cos \frac{x}{2}\right] + c$

HW

Q1 $\int \cos^3 x\, dx$

Q2 $\int \cos^4 x\, dx$

$\sin 2x = 2 \sin x \cos x$

$\sin x = 2 \sin \frac{x}{2} \cos \frac{x}{2}$

HW

very short

Q1 to Q5

Pg 1c-20

Unit - Integral Calculus.

Ch- Integration By Substitution

(1) $$\int \frac{f'(x)}{f(x)}\,dx = \log f(x) + c$$

(2) $$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

Q $$\int \frac{2x+9}{x^2+9x+30}\,dx$$

Sol $$\log(x^2+9x+30)+c$$

Q $$\int \frac{(\tan^{-1}x)^2}{1+x^2}\,dx \quad -(1)$$

Sol Put $\tan^{-1}x = t$

$$\frac{1}{1+x^2} = \frac{dt}{dx}$$

$$\frac{1}{1+x^2}\,dx = dt$$

from (1)

$$\int t^2\,dt = \frac{t^3}{3} + c$$

$$= \frac{(\tan^{-1}x)^3}{3} + c$$

Unit - Integral Calc

- Integration

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)+c$$

$$\int \frac{1}{x^2-a^2}\,dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)+c$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$$

$$\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \log\left(x+\sqrt{a^2+x^2}\right)+c$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left(x+\sqrt{x^2-a^2}\right)+c$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}+c$$

Unit - Integral Calculus.

Ch- Integration By Substitution

Formulae

$$\int \tan x \, dx = \log \sec x + c$$

$$\int \cot x \, dx = \log \sin x + c$$

$$\int \sec x \, dx = \log(\sec x + \tan x) + c$$

$$\int \operatorname{cosec} x \, dx = \log(\operatorname{cosec} x - \cot x) + c$$

HW

Q1. $\displaystyle\int \frac{\sin(\tan^{-1}x)}{1+x^2}\, dx$, Q2 $\displaystyle\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$

Q3. $\displaystyle\int \frac{\sec^2(\log x)}{x}\, dx$, Q4. $\displaystyle\int \frac{(\log x)^2}{x}\, dx$

ral Calculus:

egration By Substitution –

)+c

Q $\int \frac{dx}{\underbrace{x^2 + 6x} + 5}$

+c

Sol

$= \int \frac{dx}{\underbrace{x^2 + 6x + 9} \underbrace{- 9 + 5}}$

$2ab = 6x$
$2\cancel{x}b = 6\cancel{x}$
$b = \frac{6}{2} = 3$

)+c

$= \int \frac{dx}{(x+3)^2 - 4}$

) + c

$= \int \frac{dx}{\underbrace{(x+3)^2} - (2)^2}$

$= \frac{1}{2(2)} \log\left(\frac{x+3-2}{x+3+2}\right) + c$

$= \frac{1}{4} \log\left(\frac{x+1}{x+5}\right) + c$

by substitution

Q $\int \frac{dx}{9x^2-12x+8}$

Sol $\int \frac{dx}{9\left(x^2-\frac{12}{9}x+\frac{8}{9}\right)}$

$= \frac{1}{9}\int \frac{dx}{\left(x^2-\frac{4}{3}x+\frac{8}{9}\right)}$

$= \frac{1}{9}\int \frac{dx}{\left(x^2-\frac{4}{3}x+\frac{4}{9}-\frac{4}{9}+\frac{8}{9}\right)}$

$= \frac{1}{9}\int \frac{dx}{\left(x-\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^2}$

$= \frac{1}{9}\frac{1}{\frac{2}{3}}\tan^{-1}\left(\frac{x-\frac{2}{3}}{\frac{2}{3}}\right)+C$

$= \frac{1}{6}\tan^{-1}\left(\frac{3x-2}{2}\right)+c$

Unit - Integral Calculus.

Ch- Integration By Substitution -

Q $\int \frac{e^{\tan^{-1}x}}{1+x^2}\,dx$ — ①

Sol Put $\tan^{-1}x = t$

$$\frac{1}{1+x^2} = \frac{dt}{dx}$$

$$\frac{1}{1+x^2}\,dx = dt$$

from ①

$$= \int e^t\,dt$$

$$= e^t + c$$

$$= e^{\tan^{-1}x} + c$$

Q $\int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\,dx$ — ①

Sol Put $\sin^{-1}x = t$

$$\frac{1}{\sqrt{1-x^2}} = \frac{dt}{dx}$$

$$\frac{1}{\sqrt{1-x^2}}\,dx = dt$$

from ①

$$\int e^t\,dt = e^t + c$$

$$= e^{\sin^{-1}x} + c$$

Calculus.

...tion By Substitution -

Q $\int \frac{dx}{x^2+9}$

Sol $\int \frac{dx}{x^2+(3)^2}$

$= \frac{1}{3} \tan^{-1} \frac{x}{3} + c$

Q $\int \frac{1}{\sqrt{x^2+36}}\, dx$

c

Sol $\int \frac{1}{\sqrt{x^2+6^2}}\, dx$

$= \log\left(x + \sqrt{x^2+6^2}\right) + c$

$= \log\left(x + \sqrt{x^2+36}\right) + c$

us.

Substitution -

HW

Ex 2.2

Q1 i) - vi)

Q2 i) ii)

Q3 i) ii)

(b) **Integrals of the type :**

(i) $\int \frac{dx}{a + b\cos x + c\sin x}$ (ii) $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx$ (iii) $\int \frac{a\cos x + b\sin x + c}{d\cos x + e\sin x + f}\,dx$.

Solution : Let $I = \int \frac{dx}{a + b\cos x + c\sin x}$

This type of integral is evaluated by putting

$$\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ and } \sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

and then let $\qquad \tan \frac{x}{2} = t$

Differentiating w.r.t. x

$$\frac{1}{2}\sec^2 \frac{x}{2}\,dx = dt$$

Given integral will be reduced to irreducible quadratic *i.e.* $\int \frac{dt}{at^2 + bt + c}$.

(c) Integrals of the type :

(i) $\int \frac{dx}{a + b\cos^2 x}$ (ii) $\int \frac{dx}{a + b\sin^2 x}$ (iii) $\int \frac{dx}{a\sin^2 x + b\cos^2 x + c}$.

All the three types can be evaluated by,

(i) Dividing the numerator and denominator by $\cos^2 x$.

(ii) Then, we get integrals respectively :

(a) $\int \frac{\sec^2 x\, dx}{a\sec^2 x + b}$ or $\int \frac{\sec^2 x\, dx}{a(1+\tan^2 x)+b}$ then put $\tan x = t$.

(b) $\int \frac{\sec^2 x\, dx}{a\sec^2 x + b\tan^2 x}$ or $\int \frac{\sec^2 x\, dx}{a(1+\tan^2 x) + b\tan^2 x}$ or $\int \frac{\sec^2 x\, dx}{(a+b)\tan^2 x + a}$ then put $\tan x = t$.

(c) $\int \frac{\sec^2 x\, dx}{a\tan^2 x + b + c\sec^2 x}$ or $\int \frac{\sec^2 x\, dx}{a\tan^2 x + c(1+\tan^2 x) + b}$ or $\int \frac{\sec^2 x\, dx}{(a+c)\tan^2 x + (b+c)}$

then put $\tan x = t$.

Example 2. Evaluate the following integrals :

(*i*) $\displaystyle\int \frac{dx}{3+2\sin x+\cos x}$ (*ii*) $\displaystyle\int \frac{dx}{5\sin x+12\cos x}$

Solution : (*i*) Let $\displaystyle I = \int \frac{dx}{3+2\sin x+\cos x}$

Put $\sin x = \dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$ and $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore \qquad I = \int \frac{dx}{3+2\left(\dfrac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)+\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$$

$$= \int \frac{\sec^2 \frac{x}{2}\, dx}{2\tan^2 \frac{x}{2} + 4\tan \frac{x}{2} + 4}$$

Put $\tan \frac{x}{2} = t$

Differentiating w.r.t. x

$$\frac{1}{2}\sec^2 \frac{x}{2}\, dx = dt \quad \text{or} \quad \sec^2 \frac{x}{2}\, dx = 2dt$$

$$\therefore \quad I = \int \frac{2dt}{2t^2 + 4t + 4} = \int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 + 1} = \int \frac{dt}{(t+1)^2 + 1^2}$$

$$= \frac{1}{1} \cdot \tan^{-1}\left(\frac{t+1}{1}\right) + C = \tan^{-1}(t+1) + C$$

$$\therefore \quad I = \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + C \textbf{ Ans.}$$

(ii) Let $I = \int \frac{dx}{5\sin x + 12\cos x}$

Put $\sin x = \frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$ and $\cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

$$\therefore \quad I = \int \frac{dx}{5\left(\frac{2\tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right) + 12\left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}\right)}$$

$$= \int \frac{\sec^2 \frac{x}{2}\, dx}{10\tan \frac{x}{2} + 12 - 12\tan^2 \frac{x}{2}}$$

(ii) Let
$$I = \int \frac{dx}{4\sin^2 x + 5\cos^2 x}$$

Dividing numerator and denominator by $\cos^2 x$, we get
$$I = \int \frac{\sec^2 x\, dx}{4\tan^2 x + 5}$$

Put
$$\tan x = t$$

Differentiating w.r.t. x
$$\sec^2 x\, dx = dt$$

$$\therefore \quad I = \int \frac{dt}{4t^2 + 5} = \frac{1}{4}\int \frac{dt}{t^2 + \left(\frac{\sqrt{5}}{2}\right)^2}$$

$$= \frac{1}{4} \cdot \frac{1}{\frac{\sqrt{5}}{2}} \cdot \tan^{-1}\left(\frac{t}{\frac{\sqrt{5}}{2}}\right) + C = \frac{1}{2\sqrt{5}} \tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + C \text{ Ans.}$$

Example 1. Integrate the following functions :

(i) $\int \frac{dx}{5+4\cos x}$ (ii) $\int \frac{dx}{5-3\cos x}$ (iii) $\int \frac{dx}{4+5\sin x}$.

Solution : (i) Let $\quad I = \int \frac{dx}{5+4\cos x}$

Put $\cos x = \dfrac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$, we get

$$I = \int \frac{dx}{5+4\left[\dfrac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}\right]} = \int \frac{dx}{\dfrac{5\left(1+\tan^2 \frac{x}{2}\right)+4\left(1-\tan^2 \frac{x}{2}\right)}{1+\tan^2 \frac{x}{2}}}$$

$$I = \int \frac{\sec^2 \frac{x}{2}\, dx}{9+\tan^2 \frac{x}{2}} \qquad \left[\because 1+\tan^2 \frac{x}{2} = \sec^2 x\right]$$

Now, put $\quad \tan \frac{x}{2} = t$

Differentiating w.r.t. x

$$\frac{1}{2}\sec^2 \frac{x}{2}\, dx = dt \text{ or } \sec^2 \frac{x}{2}\, dx = 2dt$$

$$\Rightarrow \quad I = \int \frac{2dt}{t^2+9} = 2\int \frac{dt}{t^2+3^2} = 2\cdot\frac{1}{3}\tan^{-1}\left(\frac{t}{3}\right) + C$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{\tan \frac{x}{2}}{3}\right) + C = \frac{2}{3}\tan^{-1}\left[\frac{1}{3}\tan \frac{x}{2}\right] + C \text{ Ans.}$$