3/4/20

Integration By Parts

$$\int uw\,dx = u\int w\,dx - \int\left[\frac{du}{dx}\int w\,dx\right]dx$$

ILATE — Exponential

Inverse Log Alg Trigono

Q $\int \underset{\text{I}}{x}\ \underset{\text{II}}{\sin x}\,dx$

Sol $x\int \sin x\,dx - \int\left[\frac{d(x)}{dx}\int \sin x\,dx\right]dx$

$= x(-\cos x) - \int\left[1(-\cos x)\right]dx$

$= -x\cos x + \int \cos x\,dx$

$= -x\cos x + \sin x + c$

13/4/20

INTEGRATION BY PARTS

$$\int uw\,dx = u\int w\,dx - \int\left[\frac{du}{dx}\int w\,dx\right]dx$$

ILATE — Exponential

Inverse Log Alg Trigono

Q $\int \underset{\mathrm{I}}{x^2}\,\underset{\mathrm{II}}{\cos x}\,dx$

Sol $x^2 \sin x - \int 2x \sin x\,dx$

$= x^2 \sin x - 2\int \underset{\mathrm{I}}{x}\,\underset{\mathrm{II}}{\sin x}\,dx$

$= x^2 \sin x - 2\left[x(-\cos x) - \int 1(-\cos x)\,dx\right]$

$= x^2 \sin x - 2\left[-x\cos x + \int \cos x\,dx\right]$

$= x^2 \sin x - 2\left[-x\cos x + \sin x\right] + c$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$

INTEGRATION BY PARTS

$$\int u w \, dx = u \int w \, dx - \int \left[\frac{du}{dx} \int w \, dx \right]$$

ILATE — Exponential

Inverse Log Alg Trigono

Q PT. $\displaystyle\int \frac{x e^x}{(x+1)^2} = \frac{e^x}{x+1}$

Sol

$$\int \frac{e^x(x+1-1)}{(x+1)^2} \, dx$$

$$= \int e^x \frac{(x+1)}{(x+1)^2} \, dx - \int \frac{e^x}{(x+1)^2} \, dx$$

$$= \int \frac{e^x}{(x+1)} \, dx - \int \frac{e^x}{(x+1)^2} \, dx$$

$$= \int \underset{\text{II}}{e^x} \cdot \underset{\text{I}}{\frac{1}{x+1}} \, dx - \int \frac{e^x}{(x+1)^2} \, dx$$

$$= \frac{1}{x+1} e^x - \int \frac{-1}{(x+1)^2} e^x - \int \frac{e^x}{(x+1)^2} \, dx$$

$$= \frac{e^x}{x+1} + \int \frac{1 \, e^x}{(x+1)^2} \, dx - \int \frac{e^x}{(x+1)^2} \, dx = \frac{e^x}{x+1}$$

13/4/20

INTEGRATION BY PARTS

$$\int uw\,dx = u\int w\,dx - \int\left[\frac{du}{dx}\int w\,dx\right]dx$$

ILATE — Exponential

Inverse Log Alg Trigono

Q $\int \underset{\text{II}}{1} . \underset{\text{I}}{\log x}\, dx$

Sol $\log x . x - \int \frac{1}{\cancel{x}} \cancel{x}\, dx$

$= x . \log x - \int 1\, dx$

$= x . \log x - x + c$

Q

Q

Q

Q

13/4/20

Integration By Parts

$$\int uw\,dx = u\int w\,dx - \int\left[\frac{du}{dx}\int w\,dx\right]dx$$

ILATE — Exponential

Inverse Log Alg Trigono

Q $\int \underset{II}{1}.\underset{I}{\sin^{-1}} x\,dx$

Sol Integrating by parts,

$= \sin^{-1}x \int 1\,dx - \int\left[\frac{d}{dx}(\sin^{-1}x)\int 1\,dx\right]dx$

$= \sin^{-1}x \cdot x - \int \frac{1}{\sqrt{1-x^2}}\,x\,dx$

$= x\sin^{-1}x - \int \frac{x}{(1-x^2)^{1/2}}\,dx$

$= x\sin^{-1}x - \int x\,(1-x^2)^{-1/2}\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int(-2x)\,(1-x^2)^{-1/2}\,dx$

$= x\sin^{-1}x + \frac{1}{2}\,\frac{(1-x^2)^{-1/2+1}}{-\frac{1}{2}+1} = x\sin^{-1}x + \sqrt{1-x^2} + c$

$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x$

$\int[f(x)]^n f'(x)\,dx$

$= \frac{f(x)^{n+1}}{n+1}$

HW

Q1. $\int \tan^{-1} x \, dx$

Q2. $\int x^2 \tan^{-1} x \, dx$

Q3. $\int \cos^{-1} x \, dx$

Q4. $\int x e^{2x} \, dx$

Q5. $\int x \log x \, dx$

Q6. $\int \frac{x^2 - x + 7}{x^2 - 3x + 2} \, dx$

Q7. $\int \frac{(x^2+4)}{(x^2+1)(x^2+3)} \, dx$

5/4/20

INTEGRATION BY PARTS

$$\int uw\,dx = u\int w\,dx - \int\left[\frac{du}{dx}\int w\,dx\right]dx$$

I L A T E — Exponential

Inverse Log Alg Trigono

Q $\int \frac{dx}{x(x+1)}$

Sol Let $\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$ — 1

Multiply b.s. by $x(x+1)$

$1 = A(x+1) + B(x)$ — 2

Put $x+1=0$ in 2 $\Rightarrow x=-1$

$1 = B(-1)$

$\boxed{B=-1}$

Put $x=0$ in 2

$1 = A(1) + B(0)$

$\boxed{A=1}$

Q2. $\int x^2 \tan^{-1} x \, dx$

Q3. $\int \cos^{-1} x \, dx$

Q4. $\int x e^{2x} \, dx$

Q5. $\int x \log x \, dx$

Put in ①

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

$$\int \frac{1}{x(x+1)} = \int \frac{1}{x} dx - \int \frac{1}{x+1} dx$$

$$= \log x - \log (x+1)$$

$$= \log \frac{x}{x+1} + c$$

14/4/20

Definite Integral

$$\boxed{\begin{aligned}\int_a^b f(x)\,dx &= \phi(x)\Big]_a^b \\ &= \phi(b)-\phi(a)\end{aligned}}$$

Q $\displaystyle\int_a^b x^n\,dx$

Sol $\displaystyle\frac{x^{n+1}}{n+1}\Big]_a^b$

$\displaystyle\frac{1}{n+1}\left[b^{n+1}-a^{n+1}\right]$

Q $\displaystyle\int_0^1 e^x\,dx$

Sol $e^x\Big]_0^1$

$= e^1 - e^0$

$= e - 1$

Q $\displaystyle\int_{\pi/6}^{\pi/2} \sin^3 x\,dx$

$\sin 3x = 3\sin x - 4\sin^3 x$

$\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)$

Sol $\displaystyle\frac{1}{4}\int_{\pi/6}^{\pi/2}(3\sin x - \sin 3x)\,dx$

$\displaystyle=\frac{1}{4}\left[\int_{\pi/6}^{\pi/2} 3\sin x\,dx - \int_{\pi/6}^{\pi/2}\sin 3x\,dx\right]$

$\displaystyle=\frac{1}{4}\left[3(-\cos x)_{\pi/6}^{\pi/2} - \frac{(-\cos 3x)}{3}\Big]_{\pi/6}^{\pi/2}\right]$

$\displaystyle=\frac{1}{4}\left(3\left(0+\frac{\sqrt{3}}{2}\right)+(0-0)\right)$

$\displaystyle=\frac{1}{4}\left[\frac{3\sqrt{3}}{2}\right] = \frac{3\sqrt{3}}{8}$

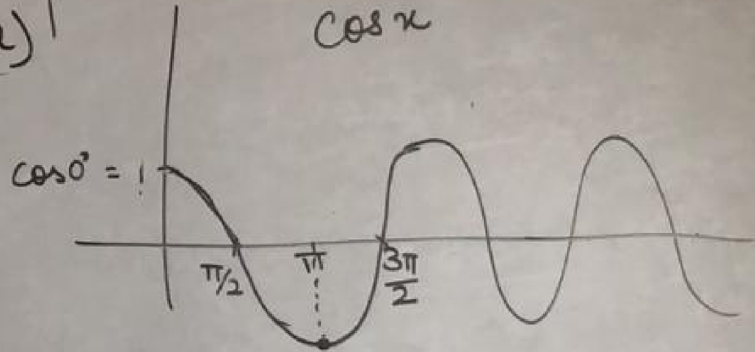
$3\left(\frac{\pi}{6}\right)$

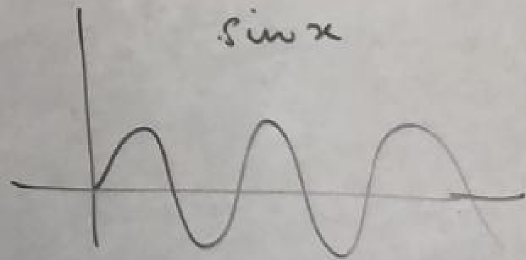
$=\frac{\pi}{2}$

14/4/20

Definite Integral

$$\boxed{\int_a^b f(x)\,dx = \phi(x)\Big]_a^b = \phi(b) - \phi(a)}$$





Q $\int_0^{\pi} |\cos x|\,dx$

Sol

$$= \int_0^{\pi/2} |\cos x|\,dx + \int_{\pi/2}^{\pi} |\cos x|\,dx$$

$$= \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} -\cos x\,dx$$

$$= \sin x\Big]_0^{\pi/2} - \sin x\Big]_{\pi/2}^{\pi}$$

$$= \sin\frac{\pi}{2} - \sin 0 - \left[\sin\pi - \sin\frac{\pi}{2}\right]$$

$$= (1-0) - (0-1) = 2$$

1/4/20

Definite Integral.

$$\boxed{\int_a^b f(x)\,dx = \phi(x)\Big]_a^b = \phi(b) - \phi(a)}$$

Q $\displaystyle\int_1^{\sqrt{3}} \frac{dx}{1+x^2}$

Sol $= \tan^{-1}x\Big]_1^{\sqrt{3}}$

$= \tan^{-1}\sqrt{3} - \tan^{-1}(1)$

$= \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\pi}{12}$

Q $\displaystyle\int_0^{\sqrt{3}} \frac{e^{m\tan^{-1}x}}{1+x^2}\,dx$ —①

Sol Put $\tan^{-1}x = t$ ✓

$\frac{1}{1+x^2} = \frac{dt}{dx}$

$\frac{dx}{1+x^2} = dt$

When $x=0$, $t=0$

When $x=\sqrt{3}$, $t=\pi/3$

from ①

$\displaystyle\int_0^{\pi/3} e^{mt}\,dt = \frac{e^{mt}}{m}\Big]_0^{\pi/3}$

$= \frac{1}{m}\left[e^{m\pi/3} - e^{m0}\right]$

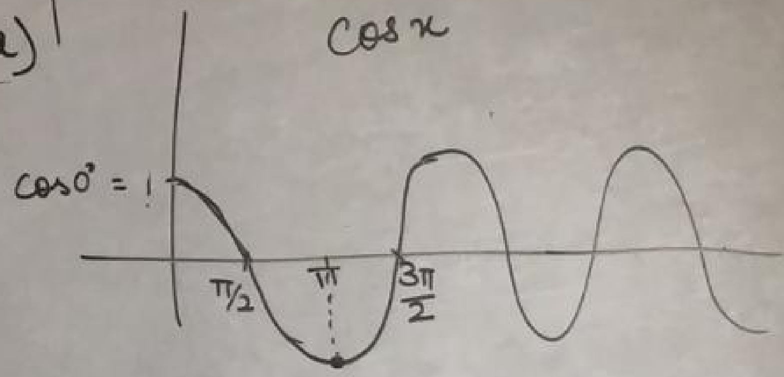
$= \frac{1}{m}\left[e^{m\pi/3} - 1\right]$

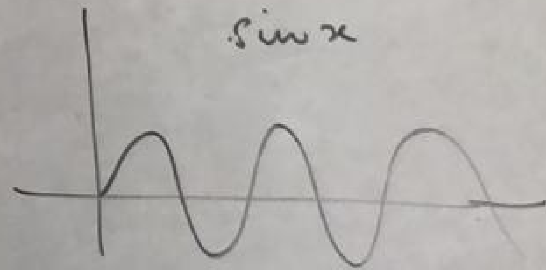
4/20

Definite Integral

$$\boxed{\begin{aligned}\int_a^b f(x)\,dx &= \phi(x)\Big]_a^b \\ &= \phi(b) - \phi(a)\end{aligned}}$$

Q $$\int_0^{\pi} |\cos x|\,dx$$





Sol

$$= \int_0^{\pi/2} |\cos x|\,dx + \int_{\pi/2}^{\pi} |\cos x|\,dx$$

$$= \int_0^{\pi/2} \cos x\,dx + \int_{\pi/2}^{\pi} -\cos x\,dx$$

$$= \sin x\Big]_0^{\pi/2} - \sin x\Big]_{\pi/2}^{\pi}$$

$$= \sin\frac{\pi}{2} - \sin 0 - \left[\sin\pi - \sin\frac{\pi}{2}\right]$$

$$= (1-0) - (0-1) = 2$$

Definite Integral

$$\boxed{\ldots)\,dx = \phi(x)\Big]_a^b = \phi(b) - \phi(a)}$$

Q6. $\displaystyle\int_0^{1/2} \frac{(\sin^{-1} x)^3}{\sqrt{1-x^2}}\,dx$

HW

Q1. PT $\displaystyle\int_0^{\pi} \frac{dx}{5+4\cos x} = \frac{\pi}{3}$

Q2. $\displaystyle\int_0^{\pi/2} \cos^2 x\,dx$

Q3. $\displaystyle\int_0^{\pi} x\cos 2x\,dx$

Q4. $\displaystyle\int_{-\pi/2}^{\pi/2} |\sin x|\,dx$

Q5 $\displaystyle\int_1^{2} x e^{2x}\,dx$

4.4. TWO STANDARD FORMULAE

(i) **Rule to evaluate $\int_0^{\pi/2} \sin^n x\,dx$ or $\int_0^{\pi/2} \cos^n x\,dx$, where n is an +ve integer.**

Case I. When n is odd positive integer.

$$\left.\begin{aligned}
&(i) \quad \int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)(n-3)(n-5)\ldots\ldots(+\text{ve factor})}{n(n-2)(n-4)\ldots\ldots\ldots\ldots(+\text{ve factor})}\\
&(ii) \quad \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\ldots\ldots(+\text{ve factor})}{n(n-2)(n-4)\ldots\ldots\ldots\ldots(+\text{ve factor})}
\end{aligned}\right\}$$

Case II. When n is even positive integer.

$$\left.\begin{aligned}
&(i) \quad \int_0^{\pi/2} \sin^n x\,dx = \frac{(n-1)(n-3)(n-5)\ldots\ldots(+\text{ve factor})}{n(n-2)(n-4)\ldots\ldots\ldots\ldots(+\text{ve factor})} \times \frac{\pi}{2}\\
&(ii) \quad \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\ldots\ldots(+\text{ve factor})}{n(n-2)(n-4)\ldots\ldots\ldots\ldots(+\text{ve factor})} \times \frac{\pi}{2}
\end{aligned}\right\}$$

Hence to write (X) and (Y), the rule is

$$\int_0^{\pi/2} \sin^n x\,dx \text{ or } \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)[\text{go on diminishing by 2 so long as factors are +ve}]}{(n)[\text{go on diminishing by 2 so long as factors are +ve}]}$$

and multiply this by $\frac{\pi}{2}$ only if n is even, each series of factors being continued so lon factors are positive.

Note. The limits of integration must be 0 to $\frac{\pi}{2}$.

(ii) ... $\int_0^{\pi/2} \sin^m x \cos^n x\,dx$, where m and n are positive integers

Case I. When either 'm' or 'n' or both are odd, then

$$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{[(m-1)(m-3)\ldots\ldots][(n-1)(n-3)\ldots.]}{[(m+n)(m+n-2)(m+n-4)\ldots.]} \quad \ldots(X)$$

Case II. When both 'm' and 'n' are even, then

$$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{[(m-1)(m-3)\ldots.][(n-1)(n-3)]}{(m+n)(m+n-2)(m+n-4)\ldots\ldots} \times \frac{\pi}{2} \quad \ldots(Y)$$

Hence, rule to remember the results (X) and (Y) is

$$\int_0^{\pi/2} \sin^m x \cos^n x\,dx = \frac{\begin{bmatrix} (m-1)\times \text{go on diminishing by 2 so long as factors are + ve} \\ \times(n-1)\times \text{go on diminishing by 2 so long as factors are + ve} \end{bmatrix}}{[(m+n)\times \text{go on diminishing by 2 so long as factors are + ve}]}$$

and multiply this by $\frac{\pi}{2}$ only if both 'm' and 'n' are even integers, each series being continued so long as the factors are +ve.

Example 20. Evaluate (i) $\int_0^{\pi/2} \sin^9 x\,dx$

(ii) $\int_0^{\pi/2} \cos^8 x\,dx$ (iii) $\int_0^{\pi/2} \sin^5 x \sin^7 x\,dx$.

Sol. (i) Let, $I = \int_0^{\pi/2} \sin^9 x\,dx$. Comparing it with $\int_0^{\pi/2} \sin^n x\,dx$.

$\therefore$ $n = 9$ which is odd. So apply the rule of 5·2(i) case I.

Hence, $\int_0^{\pi/2} \sin^9 x\,dx = \frac{8\cdot6\cdot4\cdot2}{9\cdot7\cdot5\cdot3\cdot1} = \frac{128}{315}$

(ii) $\int_0^{\pi/2} \cos^8 x\,dx$; Comparing it with $\int_0^{\pi/2} \cos^n x\,dx$.

Here $n = 8$ which is even. So apply the rule 5·2(i) case II.

Hence, $\int_0^{\pi/2} \cos^8 x\,dx = \frac{7\cdot5\cdot3\cdot1}{8\cdot6\cdot4\cdot2} \times \frac{\pi}{2} = \frac{35\pi}{256}$

xample 24. Evaluate $\int_0^{\pi/6} \cos^5 3x\, dx$.

ol. Let, $$I = \int_0^{\pi/6} \cos^5 3x\, dx$$

'ut $3x = t$ and differentiate, we get, $3dx = dt$ or $dx = \dfrac{dt}{3}$.

Vhen $x = 0$, $t = 3 \times 0 = 0$ and when $x = \pi/6, t = 3 \times \pi/6 = \pi/2$.

∴ $I = \int_0^{\pi/2} \cos^5 t \dfrac{dt}{3} = \dfrac{1}{3}\int_0^{\pi/2} \cos^5 t\, dt$; Here $n = 5$ which is odd.

∴ $$I = \int_0^{\pi/6} \cos^5 3x\, dx = \frac{1}{3}\int_0^{\pi/2} \cos^5 t\, dt = \frac{1}{3} \cdot \frac{4 \cdot 2}{5 \cdot 3 \cdot 1} = \frac{8}{45}$$ (Ans

Example 21. Evaluate :

(a) $\int_0^{\pi/2} \sin^5 x \cos^7 x \, dx$ *(b)* $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$

Sol. *(a)* Let, $I = \int_0^{\pi/2} \sin^5 x \cos^7 x \, dx,$

Comparing it with $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$; We have, $m = 5, n = 7$ (both odd)

$\therefore$ $I = \frac{[4 \cdot 2][6 \cdot 4 \cdot 2]}{12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2} = \frac{1}{120}$ applying the rule of 5·2*(ii)* case I

Hence $\int_0^{\pi/2} \sin^5 x \cos^7 x \, dx = \frac{1}{120}$ **(Ans.)**

(b) Let, $I = \int_0^{\pi/2} \sin^6 x \cos^5 x \, dx$. Comparing it with $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$

Here $m = 6$ (even) and $n = 5$ (odd). Applying the rule.

$\therefore$ $I = \frac{[5 \cdot 3 \cdot 1][4 \cdot 2]}{11 \cdot 9 \cdot 7 \cdot 5 \cdot 3 \cdot 1} = \frac{8}{693}$

Hence $\int_0^{\pi/2} \sin^6 x \cos^5 x \, dx = \frac{8}{693}$

Example 22. Evaluate : $\int_0^{\pi/2} \sin^6 x \cos^2 x \, dx$

Sol. Let, $I = \int_0^{\pi/2} \sin^6 x \cos^2 x \, dx$. Comparing it with $\int_0^{\pi/2} \sin^m x \cos^n x \, dx$

Here $m = 6, n = 2$ both are even. $\therefore$ applying the rule.

We get, $I = \frac{5 \cdot 3 \cdot 1 \cdot 1}{8 \cdot 6 \cdot 4 \cdot 2} \times \frac{\pi}{2}$

Hence $\int_0^{\pi/2} \sin^6 x \cos^2 x \, dx = \frac{5\pi}{256}$.

Example 23. Evaluate $\int_0^{\pi/6} \sin^4 6\theta \cos^3 3\theta \, d\theta$.

Sol. Let, $I = \int_0^{\pi/6} \sin^4 6\theta \cos^3 3\theta \, d\theta = \int_0^{\pi/6} (2 \sin 3\theta \cos 3\theta)^4 \cos^3 3\theta \, d\theta$

$$= 16\int_0^{\pi/6} \sin^4 3\theta \cos^4 3\theta \cos^3 3\theta \, d\theta = 16\int_0^{\pi/6} \sin^4 3\theta \cos^7 3\theta \, d\theta$$

Put $3\theta = x$; $\therefore$ $3d\theta = dx$ or $d\theta = \frac{1}{3}dx$

When $\theta = 0$, $x = 0$ and when $\theta = \frac{\pi}{6}$, $x = \frac{\pi}{2}$

$$\therefore \quad I = 16\int_0^{\pi/2} \sin^4 x \cos^7 x \frac{1}{3} dx = \frac{16}{3}\int_0^{\pi/2} \sin^4 x \cos^7 x \, dx$$

$$= \frac{16}{3} \cdot \frac{3 \cdot 1 \cdot 6 \cdot 4 \cdot 2}{11 \cdot 9 \cdot 7 \cdot 5 \cdot 3 \cdot 1} = \frac{256}{3456} \text{ (Ans.)}$$

Example 24. Evaluate $\int_0^{\pi/6} \cos^5 3x \, dx.$

Sol. Let, $I = \int_0^{\pi/6} \cos^5 3x \, dx$

Put $3x = t$ and differentiate, we get, $3dx = dt$ or $dx = \frac{dt}{3}$.

When $x = 0$, $t = 3 \times 0 = 0$ and when $x = \pi/6$, $t = 3 \times \pi/6 = \pi/2$.

$$\therefore \quad I = \int_0^{\pi/2} \cos^5 t \frac{dt}{3} = \frac{1}{3}\int_0^{\pi/2} \cos^5 t \, dt \text{ ; Here } n = 5 \text{ which is odd.}$$

$$\therefore \quad I = \int_0^{\pi/6} \cos^5 3x \, dx = \frac{1}{3}\int_0^{\pi/2} \cos^5 t \, dt = \frac{1}{3} \cdot \frac{4 \cdot 2}{5 \cdot 3 \cdot 1} = \frac{8}{45} \text{ (Ans.)}$$

HW

Eg 20 - 24
(all parts)

Ch-5 Applications of Integration

Area $= \int_a^b y \, dx$ $\downarrow$ x-axis

Area $= \int_c^d x \, dy$ $\downarrow$ y-axis

Q

$y = \log x$

b/w x-axis & $x = 2$ & $x = 3$

Sol

$$\text{Area} = \int_a^b y \, dx$$

$$= \int_2^3 \underset{\text{II}}{1} \cdot \underset{\text{I}}{\log x} \, dx$$

$$= \log x \cdot x - \int \frac{1}{x} \times x \Big|_2^3$$

$$= x \log x - x \Big]_2^3$$

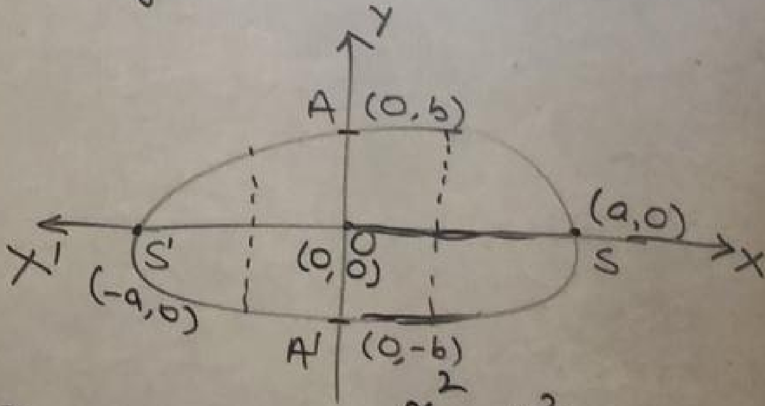
$$= 3 \log 3 - 3 - 2 \log 2 + 2 = 3 \log 3 - 2 \log 2 - 1 \text{ sq units}$$

Ch-5 Applications of Integration

$$\text{Area} = \int_a^b y\,dx$$
$\downarrow$
x - axis

$$\text{Area} = \int_c^d x\,dy$$
$\downarrow$
y - axis

Q $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Sol $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \checkmark$ — (1)

b/w $x = 0$ + $x = a$

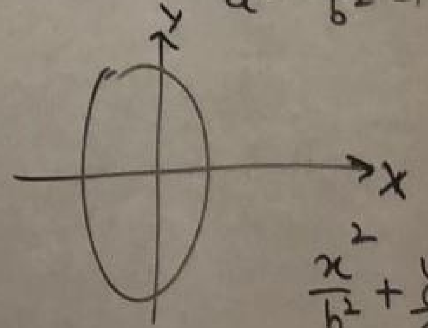
Sol $\text{Area} = 4\int_0^a y\,dx$ — (2)

from (1)

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2 - x^2}{a^2}$$

$$y^2 = \frac{b^2}{a^2}(a^2 - x^2)$$

$$y = \frac{b}{a}\sqrt{a^2 - x^2}$$



$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

Put in (2)

$$\text{Area} = \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\,dx = 4\frac{b}{a}\left|\frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right|_0^a = 4\frac{b}{2}a\sin^{-1}1$$

$$= 4\frac{ba}{2}\frac{\pi}{2} = \pi ab$$

sq units

Ch-5 Applications of Integration

$$\text{Vol} = \int_a^b \pi y^2 \, dx$$

$\downarrow$

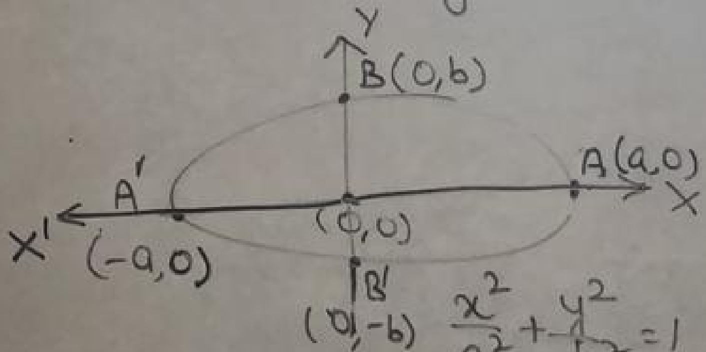
x - axis

$$\text{Vol} = \int_c^d \pi x^2 \, dy$$

$\downarrow$

y- axis

Q $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$



$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$V = \int_{-a}^{a} \pi y^2 \, dx$$

$$= \pi \int_{-a}^{a} \frac{b^2}{a^2}(a^2 - x^2)\, dx$$

$$= \frac{\pi b^2}{a^2} \int_{-a}^{a} (a^2 - x^2)\, dx = \frac{\pi b^2}{a^2}\left[a^2 x - \frac{x^3}{3}\right]_{-a}^{a}$$

$$= \frac{\pi b^2}{a^2}\left[a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3}\right]$$

$$= \frac{4\pi a b^2}{3} \text{ cu. units}$$

HW

Exercise 5.1

Q1, 2

Very short Q1 to Q4

Eg 3, 7, 8, 9

Applications of Integration

$$\text{Area} = \int_c^d \underset{\substack{\downarrow \\ y\text{-axis}}}{x\, dy}$$

Q $y = 2 + x$ $y - x = 2$

Sol $\text{Area} = \int_a^b y\, dx$

$y = 0 \Rightarrow 0 = 2 + x$

$x = -2$

$$\text{Area} = \int_0^{-2} (2+x)\,dx$$

$$= 2x + \frac{x^2}{2}\Big]_0^{-2}$$

$$= 2(-2) + \frac{4}{2} - 0 = -4 + 2 = -2$$

Ch-5 Applications of Integration

$$\text{Area} = \int_a^b y\,dx \quad \downarrow \quad x\text{-axis}$$

$$\text{Area} = \int_c^d x\,dy \quad \downarrow \quad y\text{-axis}$$

Q $y^2 = 4ax$

$y = 2\sqrt{ax}$

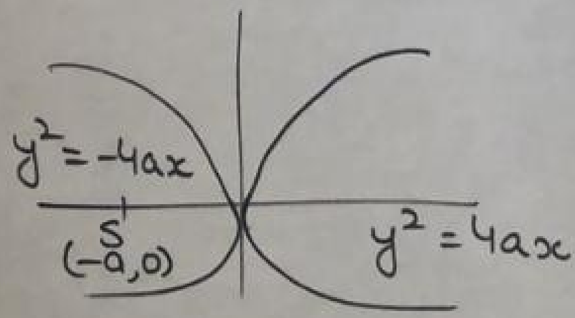
$$\text{Area} = 2\int_0^a y\,dx$$

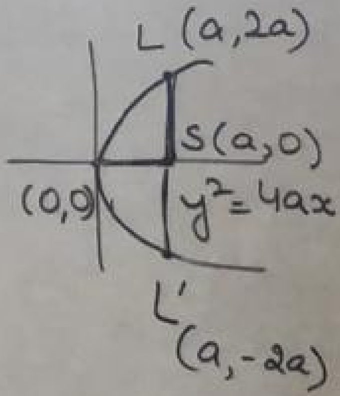
$$= 2\times 2\int_0^a \sqrt{a}\sqrt{x}\,dx$$

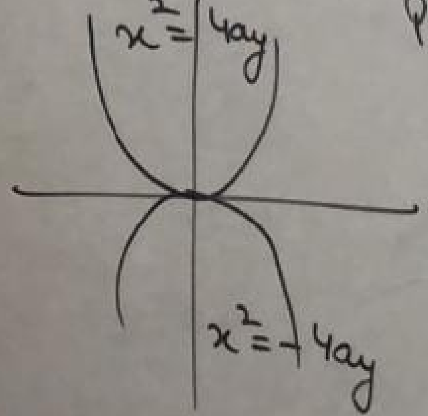
$$= 4\sqrt{a}\int_0^a x^{1/2}\,dx$$

$$= 4\sqrt{a}\left.\frac{x^{3/2}}{\frac{3}{2}}\right]_0^a$$

$$= 4\sqrt{a}\,\frac{2}{3}\left[a^{3/2}-0\right] = \frac{8a^2}{3}$$

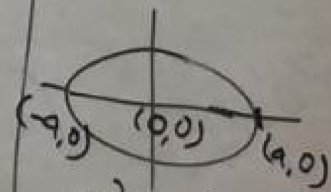






Parabola

Ellipse.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Ch- Numerical Integration or Approximate Integration

$$\int_a^b f(x)\,dx$$

Trapezoidal Rule

Area $= \frac{h}{2}\Big[$(Ist ordinate + Last ordinate) + 2(Sum of rem ordinates)$\Big]$

$$h = \frac{b-a}{n}$$

Q $\int_0^7 x^2\,dx$ using 8 ordinates

Sol

$n = 7$

$$h = \frac{7-0}{7} = 1$$

$x =$	0	1	2	3	4	5	6	7
$y = x^2$	0	1	4	9	16	25	36	49
	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

$$\text{Area} = \frac{h}{2}\left[(y_1 + y_8) + 2(y_2 + y_3 + y_4 + y_5 + y_6 + y_7)\right]$$

$$= \frac{1}{2}\left[(0+49) + 2(1+4+9+16+25+36)\right]$$

$$= \frac{1}{2}\left[49 + 2(91)\right]$$

$$= \frac{1}{2}\left[49 + 182\right]$$

$$= \frac{231}{2} = 115.5 \text{ sq units}$$

Ch- Numerical Integration or Approximate Integration

$$\int_a^b f(x)\,dx$$

Trapezoidal Rule.

Area $= \frac{h}{2}\left[(\text{Ist ordinate} + \text{Last ordinate}) + 2(\text{Sum of rem. ordinates})\right]$

$$h = \frac{b-a}{n}$$

Q

x	0	1	2	3	4	5
y	0	2.5	3	4.5	5	7.5
	y_1	y_2	y_3	y_4	y_5	y_6

Sol.

$$\int_0^5 y\,dx \qquad h = \frac{5-0}{5} = \frac{5}{5} = 1$$

$$\text{Area} = \frac{1}{2}\left[(0+7.5) + 2(2.5+3+4.5+5)\right]$$

$$= \frac{1}{2}\left[7.5 + 2(15)\right]$$

$$= \frac{1}{2}\left[7.5+30\right] = 18.75 \text{ sq units}$$

Ch- Numerical Integration or
Approximate Integration

$$\int_a^b f(x)\,dx$$

Simpson's Rule

Area $= \frac{h}{3}$ [(Ist ordinate + Last ordinate) + 2(Sum. of rem. odd ordinates) + 4(Sum of even ordinates)]

Q $\int_{-3}^{3} x^6\,dx$, 7 ordinates.

Sol: $h = \frac{3+3}{6} = \frac{6}{6} = 1$

x	-3	-2	-1	0	1	2	3
$y = x^6$	729	64	1	0	1	64	729
	y_1	y_2	y_3	y_4	y_5	y_6	y_7

$$\text{Area} = \frac{1}{3}[(729 + 729) + 2(1+1) + 4(64+0+64)]$$

$$= \frac{1}{3}[1458 + 4 + 512]$$

$$= \frac{1974}{3} = 658$$

Ch- Numerical Integration or Approximate Integration

$$\int_a^b f(x)\,dx$$

Simpson's Rule.

Area $= \frac{h}{3}$ [(Ist ordinate + Last ordinate) + 2(sum. of rem. odd ordinates) + 4(sum of even ordinates)]

Q

x	0	1	2	3	4	5	6
y	0.146	0.161	0.176	0.190	0.204	0.217	0.230
	y_1	y_2	y_3	y_4	y_5	y_6	y_7

Sol

$$\text{Area} = \frac{1}{3}\left[(0.146+0.230)+2(0.176+0.204)+4(0.161+0.190+0.217)\right]$$

$$= \frac{1}{3}\left[0.376+0.760+2.272\right]$$

$$= \frac{1}{3}\left[3.408\right] = \underline{1.136}$$

HW

Eg 1, 6

Ex 6.1

Q 2(ii), 7

Unit - Differential Equations

Ch - Differential Equations

& their formation.

$$\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 3 = 0 \rightarrow \text{Non Linear diff eqn.}$$

$$\frac{dy}{dx} = x + 3\cos x \quad - \text{Linear diff eqn}$$

		Order	Degree.
1	$\left(\frac{dy}{dx}\right)^1 = x + 3\cos x$	1	1
2	$\left(\frac{d^2y}{dx^2}\right)^1 + 5x\frac{dy}{dx} + 7y = 0$	2	1
3	$\sqrt{\frac{dy}{dx}} = 3x + \frac{dy}{dx}$ Sq b.s. $\frac{dy}{dx} = 9x^2 + \left(\frac{dy}{dx}\right)^2 + 6x\frac{dy}{dx}$	1	2

Unit - Differential Equations

Ch - Differential Equations & their formation

HW
Very Ques 1 to 5
Fillups, T/F, Multiple choice ques.

	Order	Degree	Linear / Non Linear
① $R\frac{di}{dt}+\frac{i}{c}=E\cos\omega t$	1	1	Linear
② $(y'')^2+y'+3y=x^2$	2	2	Non - linear
③ $(x+1)\frac{dy}{dx}+2y=x$	1	1	linear
④ $\left(\frac{d^2y}{dx^2}\right)^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$	2	2	Non - linear
⑤ $[(1+y_3^2)]^{4/5}=(1+y_3)$	3	5	Non-linear
⑥ $\frac{dy}{dx}+y\cos x=\sin x$	1	1	Linear
⑦ $\sqrt[4]{1+\left(\frac{d^2y}{dx^2}\right)^3}=\frac{d^3y}{dx^3}$	3	4	Non-linear

$\left(1+\frac{d^2y}{dx^2}\right)^3=\left(\frac{d^3y}{dx^3}\right)^4$

Unit - Differential Equations

Ch - Differential Equations & their formation

Q $y = x^2 + Cx \Rightarrow y = x(x+c)$

$c = \frac{y}{x} - x$

Sol Diff b.s. wrt x,

$$\frac{dy}{dx} = 2x + c$$

$$\frac{dy}{dx} = 2x + \frac{y}{x} - x$$

$$\frac{dy}{dx} = \frac{2x^2 + y - x^2}{x}$$

$$x\frac{dy}{dx} = x^2 + y$$

Unit - Differential Equations

Ch- Differential Equations & their formation

Q $y^2 = Ax^2 + Bx + C.$

Sol Diff b.s. wrt x,

$$2y\frac{dy}{dx} = 2Ax + B$$

$$y\frac{dy}{dx} = Ax + \frac{B}{2}$$

I II

Diff again wrt x,

$$y\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{dy}{dx}\right) = A(1) + 0$$

$$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = A$$

I II

Again diff wrt x,

$$y\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + 2\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right) = 0$$

Order = 3

Degree = 1

Unit - Differential Equations

Ch- Differential Equations & their formation.

Q $y = Ae^{3x} + Be^{-3x}$ — (1)

Sol Diff b.s. wrt x,

HW

Eg 1, 2, 3.

$$\frac{dy}{dx} = Ae^{3x}(3) + Be^{-3x}(-3)$$

Again diff wrt x,

$$\frac{d^2y}{dx^2} = 3Ae^{3x}(3) - 3Be^{-3x}(-3)$$

$$= 9Ae^{3x} + 9Be^{-3x}$$

$$= 9(\underbrace{Ae^{3x} + Be^{-3x}})$$

from (1)

$$\frac{d^2y}{dx^2} = 9y \text{ or } \frac{d^2y}{dx^2} - 9y = 0$$

Order = 2

Degree = 1

Unit - Differential Equations

Ch- First Order & Degree

Differential Equations + Solution

Variable Separable Method.

$$f(x)\,dx = F(y)\,dy$$

$$\int f(x)\,dx = \int F(y)\,dy$$

→ Removal of fraction.

→ Collection of terms.

→ Standard form

⇒ Integration.

Q $y\,dx - x\,dy = xy\,dx$

Sol $\dfrac{dx}{x} - \dfrac{dy}{y} = dx$

Integrating b.s.

$$\int \frac{dx}{x} - \int \frac{dy}{y} = \int dx + \log c$$

$$\log x - \log y = x + \log c$$

$$\log \frac{x}{y} = x + \log c$$

$$\log \frac{x}{y} - \log c = x$$

$$\log \frac{x}{cy} = x$$

$$\frac{x}{cy} = e^{x} \Rightarrow x = e^{x} cy$$

Unit - Differential Equations

Ch- First Order & Degree

Differential Equations + Solution

Variable separable Method.

$$f(x)\,dx = F(y)\,dy$$

$$\int f(x)\,dx = \int F(y)\,dy$$

→ Removal of fraction.

→ Collection of terms.

→ Standard form

⇒ Integration

Q $3e^x \tan y\,dx + (1+e^x)\sec^2 y\,dy = 0$

Sol Dividing b.s. by $\tan y(1+e^x)$

$$\Rightarrow \frac{3e^x}{1+e^x}dx + \frac{\sec^2 y}{\tan y}dy = 0$$

Integrating b.s.

$$3\int \frac{e^x}{1+e^x}dx + \int \frac{\sec^2 y}{\tan y}dy = c$$

$$3\log(1+e^x) + \log\tan y = c$$

$$\log(1+e^x)^3 + \log\tan y = c$$

$$\log(1+e^x)^3\tan y = c$$

$$(1+e^x)^3\tan y = e^c$$

$$(1+e^x)^3\tan y = C$$

it - Differential Equations

Ch- First Order & Degree

Differential Equations + Solution

ble Method.

(y) dy

$= \int F(y)\,dy$

action

terms

m

Q $\cancel{e^y}(dy + dx) = x\cancel{e^y}\,dx$

Sol

$dy + dx = x\,dx$

$dy = x\,dx - dx$

$dy = (x-1)\,dx$

Integrating b.s.

$\int dy = \int (x-1)\,dx$

$y = \frac{x^2}{2} - x + c$

ifferential Equations

irst Order & Degree

Differential Equations & Solution

nethod.

Q $\frac{dy}{dx} = x + \frac{1}{x} + x - x^2$

y

So: $\frac{dy}{dx} = 2x + \frac{1}{x} - x^2$

y) dy

$dy = \left(2x + \frac{1}{x} - x^2\right) dx$

Integrating b.s.

$y = \cancel{2}\frac{x^2}{\cancel{2}} + \log x - \frac{x^3}{3} + c$

$y = x^2 + \log x - \frac{x^3}{3} + c$

HW

Ex 1.1 Pg DE-29

Q 1, 3, 11, 13

Example 9, 13, 6